Performance Analysis of Nakagami-m Fading Massive MIMO Channels with Linear Receivers

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Abstract—This paper estimates bounds on the capacity of Nakagami-\(m\) fading massive multiple-input multiple-output (MIMO) channels with large number of antennas at the transmitter and receiver. We consider two models namely conventional co-located MIMO (C-MIMO) and distributed MIMO (D-MIMO) models. At first, we derive the achievable rate for the C-MIMO and D-MIMO systems. Secondly, we use maximum ratio combining (MRC), zero-forcing (ZF) and minimum mean square error (MMSE) detectors in the receiver and derive the expression of achievable rate for both the C-MIMO and D-MIMO systems. Finally, we analyze the asymptotic behavior of the ergodic capacity when the number of antennas at one or both sides goes to infinity. Our investigation concludes that the ergodic capacity of both the co-located and distributed MIMO channels is intuitive as the increasing number of antennas helps to eliminate the effect of fading. MMSE detector shows significant performance in enhancing the ergodic capacity of Nakagami-\(m\) fading channels compared to ZF and MRC detectors.

Index Terms—Asymptotic behavior, co-located MIMO, distributed MIMO.

I. INTRODUCTION

MIMO technology attracts great attention as the subject of extensive theoretical and practical investigation for the next generation wireless cellular systems [1]. The D-MIMO channel reflects the distinctive large-scale fading effects for each antenna-pair, making it useful for analyzing a MIMO channel with the antennas distributed in a large area. On the other hand, the C-MIMO channel is used for the analysis of a traditional point-to-point MIMO channel where the antennas at either side are co-located, and have the same large-scale fading.

Motivated by these issues, in this paper, we investigate the performance of MRC, ZF and MMSE receivers in enhancing the achievable rate of Nakagami-\(m\) fading C-MIMO and D-MIMO channels. Recently, the lower bound of the capacity of a MIMO network was estimated in [2] for both the perfect and imperfect channel state information (CSI) at the receiver and it was observed that in a certain operating region, it is possible to increase the achievable rate for large MIMO system using MRC, ZF and MMSE detectors at the receiver.

In [3], a simple robust approach was used to shows how the mutual information varies with the number of transmitting and receiving antennas. A mixed mode MIMO network using Time Division Duplexing (TDD) technique is introduced in [4] to compare its achievable rate with the massive MIMO architecture. The asymptotic sum capacity of an uplink flat fading MIMO system was derived in [5] using joint decoding process of all users. It was found that the achievable rate of joint coding system is larger than the system that uses single user decoding.

In this paper, at first, we derive the achievable rate for the C-MIMO and D-MIMO systems. Secondly, we use MRC, ZF and MMSE detectors in the receiver and derive the expression of achievable rate for both the C-MIMO and D-MIMO systems. Finally, we analyze the asymptotic behavior of the ergodic capacity when the number of antennas at one or both sides goes to infinity.

The remainder of the this paper is organized as follows. The system model is described in Section II. Section-III describes the formulation of achievable rate of C-MIMO channel using MRC, ZF and MMSE detectors at the receivers. Section-IV describes the formulation of achievable rate of D-MIMO channel using MRC, ZF and MMSE detectors at the receivers. Numerical results comparing the performance of MRC, ZF and MMSE detectors are provided in Section V. Finally, Section VI provides the concluding remarks of this work.

II. SYSTEM MODEL

We consider a Nakagami-\(m\) fading MIMO system with \(N_t\) transmit and \(N_r\) receive antennas as shown in Fig.1. This system can be divided into two mathematical models, i.e., C-MIMO and D-MIMO models. The C-MIMO model will be used for the analysis of a traditional point-to-point MIMO channel where the antennas at either side are co-located while the D-MIMO model reflects the distinctive large-scale fading effects for analyzing a MIMO channel with the antennas distributed in a large area. For a C-MIMO system, the \(N_t \times 1\) received signal vector \(y_{C-MIMO}\) at receiver is related with \(N_t \times 1\)

Fig. 1. System model.
transmitted signal vector \( \mathbf{x} \) by

\[
y_{C\text{-MIMO}} = \sqrt{\frac{P}{N_t}} \mathbf{H} \mathbf{x} + \mathbf{z},
\]

where \( \mathbf{H} \) is a \( N_r \times N_t \) matrix which denotes the channels co-efficient and \( \mathbf{z} \) is a \( N_r \times 1 \) vector which denotes the additive white Gaussian noise at the receiver. \( P \) denotes the total transmit power. For a D-MIMO system the received signal vector \( y_{D\text{-MIMO}} \) at the receiver is related with \( LN_t \times 1 \) transmitted signal vector \( \mathbf{x} \) by

\[
y_{D\text{-MIMO}} = \sqrt{\frac{P}{LN_t}} \mathbf{H} \mathbf{W}^{\frac{1}{2}} \mathbf{x} + \mathbf{z} = \sqrt{\frac{P}{LN_t}} \mathbf{B} \mathbf{x} + \mathbf{z},
\]

where \( \mathbf{B} = \mathbf{H} \mathbf{W}^{\frac{1}{2}} \) is the channel matrix between the \( N_r \) receive antennas and \( LN_t \) transmit antennas, \( L \) denotes the number of radio ports located at the transmitting side and \( \mathbf{W}^{\frac{1}{2}} = \text{diag}(\sqrt{\frac{1}{D_1^t}}, \ldots, \sqrt{\frac{1}{D_{L}^t}}, \ldots, \sqrt{\frac{1}{D_{N_t}^t}}) \in \mathbb{R}^{LN_t \times LN_t} \) which represents a \( LN_t \times LN_t \) diagonal matrix accounting for the large-scale fading effect, in which the path loss is characterized by \( D_{i}^t \) for some exponent \( v \), \( \{l_i\}_{i=1}^{L} \) are the independent random variables.

### III. ACHIEVABLE RATE FOR C-MIMO MODEL

In this section, we find the achievable rate for the C-MIMO with perfect channel state information at the receiver. Assume that \( \mathbf{A} \) be a \( N_r \times N_t \) linear detector matrix which depends on \( \mathbf{H} \). By using linear detector, the received signal can be separated into streams as follows:

\[
\mathbf{r} = \mathbf{A}^\dagger y_{C\text{-MIMO}}
\]

The linear detector matrix \( \mathbf{A} \) is defined as follows for the MRC, ZF and MMSE detectors:

\[
\mathbf{A} = \begin{cases} 
\mathbf{H} & \text{for MRC} \\
\mathbf{H} (\mathbf{H}^\dagger \mathbf{H})^{-1} & \text{for ZF} \\
\mathbf{H} (\mathbf{H}^\dagger + \frac{N_t}{P} \mathbf{I}_{N_t})^{-1} & \text{for MMSE}
\end{cases}
\]

From (1) and (3), the received signal vector after using linear detectors at the receiver is given by

\[
\mathbf{r} = \sqrt{\frac{P}{N_t}} \mathbf{A}^\dagger \mathbf{H} \mathbf{x} + \mathbf{A}^\dagger \mathbf{z}
\]

Let \( r_m \) and \( x_m \) be the \( m \)th elements of the \( N_t \times 1 \) vectors \( \mathbf{r} \) and \( \mathbf{x} \), respectively, where \( m = 1, 2, \ldots, N_t \). Then,

\[
r_m = \sqrt{\frac{P}{N_t}} a_m^\dagger \mathbf{H} \mathbf{x} + a_m^\dagger \mathbf{z}
\]

\[
= \sqrt{\frac{P}{N_t}} a_m^\dagger h_m x_m + \sqrt{\frac{P}{N_t}} \sum_{i=1, i \neq m}^{N_t} a_m^\dagger h_i x_i + a_m^\dagger \mathbf{z}.
\]

where \( a_m \) and \( h_m \) are the \( m \)th columns of \( \mathbf{A} \) and \( \mathbf{H} \) matrices, respectively. Therefore, the ergodic achievable rate of the \( m \)th antenna is

\[
P_m^{(C\text{-MIMO})} = \mathbb{E} \left\{ \log_2 \left( 1 + \frac{P}{N_t} |a_m^\dagger h_m|^2 / \left( \sum_{i=1, i \neq m}^{N_t} |a_i^\dagger h_i|^2 + |a_m|^2 \right) \right) \right\}
\]

#### A. MRC Receiver

In the case of MRC detectors at the receiver, we have \( \mathbf{A} = \mathbf{H} \). Hence \( a_m^\dagger = h_m \), and from (7), the achievable rate of the \( m \)th transmit antenna is given by

\[
P_m^{(C\text{-MIMO-MRC})} = \mathbb{E} \left\{ \log_2 \left( 1 + \frac{P}{N_t} |h_m|^4 / \left( \sum_{i=1, i \neq m}^{N_t} |h_i|^2 + |h_m|^2 \right) \right) \right\}
\]

For a massive MIMO channel, \( N_t \) and \( N_r \) becomes very large i.e. \( N_r, N_t \rightarrow \infty \). Therefore, in the case of massive MIMO channel, \( R_m^{(C\text{-MIMO-MRC})} \) can be derived as follows:

\[
\tilde{R}_m^{(C\text{-MIMO-MRC})} = \log_2 \left( 1 + \frac{P}{N_t} (N_r - 1) \beta_m \sum_{i=1, i \neq m}^{N_r} \beta_i + 1 \right)
\]

where \( \beta_m = \left( \frac{h_m}{\mu\zeta_i} \right)^2 \), in which \( \zeta = 1, 2, \ldots, N_r \), \( h_m \) denotes the channel coefficient between \( m \)th transmit antenna and \( \zeta \)th receive antenna, and \( \mu\zeta_i \) denotes the fast fading coefficient.

#### B. ZF Receiver

In the case of ZF detectors at the receiver, we have \( \mathbf{A}^\dagger = (\mathbf{H}^\dagger \mathbf{H})^{-1} \mathbf{H} \) or \( \mathbf{A} = \mathbf{I}_{N_t} \). Hence \( a_m^\dagger = h_m \), where \( \delta_{mi} = 1 \) when \( m = i \) and 0 otherwise. Therefore, the achievable rate for the \( m \)th transmit antenna can be derived from (7) as follows:

\[
P_m^{(C\text{-MIMO-ZF})} = \mathbb{E} \left\{ \log_2 \left( 1 + \frac{P}{N_t} |(\mathbf{H}^\dagger \mathbf{H})^{-1}|_{m,m} \right) \right\}
\]

For a massive MIMO channel, \( R_m^{(C\text{-MIMO-ZF})} \) can be derived as follows.

\[
\tilde{R}_m^{(C\text{-MIMO-ZF})} = \log_2 \left( 1 + \frac{P}{N_t} (N_r - N_t) \beta_m \right)
\]

#### C. MMSE Receiver

In the case of MMSE detectors at the receiver, we have \( \mathbf{A}^\dagger = (\mathbf{H}^\dagger \mathbf{H} + \frac{N_t}{P} \mathbf{I}_{N_t})^{-1} \mathbf{H}^\dagger = \mathbf{H}^\dagger (\mathbf{H}^\dagger \mathbf{H} + \frac{N_t}{P} \mathbf{I}_{N_t})^{-1} \). Therefore, the \( m \)th column of \( \mathbf{A} \) matrix can be written by

\[
a_m = (\mathbf{H}^\dagger + \frac{N_t}{P} \mathbf{I}_{N_t})^{-1} \mathbf{h}_m = \mathbf{A}^{-1}_m \mathbf{h}_m \mathbf{h}_m^\dagger \mathbf{A}^{-1}_m + 1.
\]
where \( \boldsymbol{A}_m = \sum_{i=1,i \neq m}^{N_t} \boldsymbol{b}_i \boldsymbol{h}_i^\dagger + \frac{N_t}{P} \boldsymbol{I}_{N_r} \). The achievable rate for the \( m \)th transmit antenna can be derived from (7) as follows:

\[
R_{m}^{(C\text{-MIMO-MMSE})} = \mathbb{E} \left\{ \log_2 \left( 1 + \frac{P}{L_{N_t}} \left| \left( \frac{\boldsymbol{I}_{N_r} + \frac{P}{N_t} \boldsymbol{H}^\dagger \boldsymbol{H} \right)^{-1} \right|_{mm} \right) \right\}
\]  

\[
R_{m}^{(C\text{-MIMO-MMSE})} = \mathbb{E} \left\{ \log_2 \left( 1 + P \right) \right\}
\]

\section*{IV. Achievable Rate for D-MIMO Model}

In this section, we find the achievable rate for the D-MIMO with perfect channel state information at the receiver. Assuming \( \mathbf{D} \) is a \( N_r \times L_{N_t} \) linear detector matrix which depends on \( \mathbf{B} \). Hence, by using linear detector, the received signal can be separated into streams as follows:

\[
\mathbf{r}_d = \mathbf{D} \mathbf{y}_{D\text{-MIMO}}
\]  

Therefore, similar to (7), the ergodic achievable uplink rate at the \( k \)th transmit antenna of D-MIMO channel can be derived as

\[
R_k^{(D\text{-MIMO})} = \mathbb{E} \left\{ \log_2 \left( 1 + \frac{P}{L_{N_t}} \left| \mathbf{d}_k \mathbf{b}_k + \frac{L_{N_t}}{P} \mathbf{I}_{L_{N_t}} \mathbf{b}_k \right|^2 \right) \right\}
\]

\[
R_k^{(D\text{-MIMO})} = \mathbb{E} \left\{ \log_2 \left( 1 + \frac{P}{L_{N_t}} \left| \mathbf{d}_k \right|^2 \right) \right\}
\]

where \( k = 1, 2, ..., L_{N_t} \), \( \mathbf{d}_k \) denotes the \( k \)th column of \( \mathbf{D} \) which depends on the channel matrix \( \mathbf{B} \) and is defined as follows:

\[
\mathbf{D} = \begin{cases} \mathbf{B} & \text{for MRC} \\ \mathbf{B} \left( \mathbf{B}^\dagger \mathbf{B} \right)^{-1} & \text{for ZF} \\ \mathbf{B} \left( \mathbf{B}^\dagger \mathbf{B} + \frac{L_{N_t}}{P} \mathbf{I}_{L_{N_t}} \right)^{-1} & \text{for MMSE} \end{cases}
\]

\section*{A. MRC Receiver}

When MRC detector is used at the receiver, then \( \mathbf{D} = \mathbf{B} \). Hence, from (14), the achievable rate of the \( k \)th transmit antenna is given by

\[
R_k^{(D\text{-MIMO-MRC})} = \mathbb{E} \left\{ \log_2 \left( 1 + \frac{P}{L_{N_t}} \left| \mathbf{b}_k \right|^4 \right) \right\}
\]

\[
R_k^{(D\text{-MIMO-MRC})} = \mathbb{E} \left\{ \log_2 \left( 1 + \frac{P}{L_{N_t}} \left| \mathbf{b}_k \right|^2 \right) \right\}
\]

For a massive MIMO channel, \( L_{N_t} \) and \( N_r \) becomes very large i.e. \( L_{N_t}, N_r \rightarrow \infty \). Therefore, in the case of massive MIMO channel, \( R_k^{(D\text{-MIMO-MRC})} \) can be derived as follows:

\[
R_k^{(D\text{-MIMO-MRC})} = \log_2 \left( 1 + \frac{P}{L_{N_t}} \left( N_r - 1 \right) \alpha_k \right) + \log_2 \left( 1 + P \alpha_k \right)
\]

\section*{B. ZF Receiver}

In the case of ZF detectors at the receiver, we have \( \mathbf{D}^\dagger = \left( \mathbf{B} \mathbf{B}^\dagger + \frac{L_{N_t}}{P} \mathbf{I}_{L_{N_t}} \right)^{-1} \mathbf{B}^\dagger \). Hence, \( \mathbf{d}_k \) for \( \mathbf{D}^\dagger, \mathbf{b}_k = \delta_{kj} \) where \( \delta_{kj} = 1 \) when \( k = j \) and 0 otherwise. Therefore, the achievable rate for the \( k \)th transmit antenna can be derived from (14) as follows:

\[
R_k^{(D\text{-MIMO-ZF})} = \mathbb{E} \left\{ \log_2 \left( 1 + \frac{P}{L_{N_t}} \left| \left( \mathbf{B} \mathbf{B}^\dagger \right)^{-1} \mathbf{b}_k \right|^2 \right) \right\}
\]

\[
R_k^{(D\text{-MIMO-ZF})} = \mathbb{E} \left\{ \log_2 \left( 1 + P \right) \right\}
\]

\section*{C. MMSE Receiver}

In the case of MMSE detectors at the receiver, we have \( \mathbf{D}^\dagger = \left( \mathbf{B} \mathbf{B}^\dagger + \frac{L_{N_t}}{P} \mathbf{I}_{L_{N_t}} \right)^{-1} \mathbf{B}^\dagger = \left( \mathbf{B} \mathbf{B}^\dagger \right)^{-1} \mathbf{B}^\dagger \mathbf{I}_{L_{N_t}} \). Hence, \( \mathbf{d}_k \) for \( \mathbf{D}^\dagger, \mathbf{b}_k = \delta_{kj} \) where \( \delta_{kj} = 1 \) when \( k = j \) and 0 otherwise. Therefore, the achievable rate for the \( k \)th transmit antenna can be derived from (14) as follows:

\[
R_k^{(D\text{-MIMO-MMSE})} = \mathbb{E} \left\{ \log_2 \left( 1 + \frac{P}{L_{N_t}} \left| \left( \mathbf{B} \mathbf{B}^\dagger \right)^{-1} \mathbf{b}_k \right|^2 \right) \right\}
\]

\[
R_k^{(D\text{-MIMO-MMSE})} = \mathbb{E} \left\{ \log_2 \left( 1 + P \right) \right\}
\]
Fig. 2. The comparison of ergodic capacity for Nakagami-
$m$ fading C-MIMO and D-MIMO channels for selected values of \( L \).

Fig. 3. The comparison of ergodic capacity for Nakagami-
$m$ fading C-MIMO and D-MIMO channels for selected values of \( L \) with MRC detector at the receiver and \( N_t = 4 \).

Fig. 4. The comparison of ergodic capacity for Nakagami-
$m$ fading C-MIMO and D-MIMO channels for selected values of \( L \) with MMSE detector at the receiver and \( N_t = 4 \).

Fig. 5. The ergodic capacity of Nakagami-
$m$ fading D-MIMO channels for MRC, ZF and MMSE detectors at the receiver and \( N_t = 4 \).

VI. CONCLUSION

In this paper, we investigate the performance of MRC, ZF and MMSE detectors in enhancing the ergodic capacity of Nakagami-
$m$ fading C-MIMO and D-MIMO channels. Based on our formulation and from the observation of numerical results, we conclude that MMSE detector is the most significant detector in enhancing the performance of Nakagami-
$m$ fading channels compared to the other linear detectors such as ZF and MRC. Although MRC has the additional benefit of facilitating a distributed per-antenna implementation of the detector but ZF outperforms MRC due to its ability to cancel out interference created between the antennas of transmitter and receiver. Finally, we observe that the use of large antenna arrays at the receiver enables us to reduce the transmit power. It is also observed that the ergodic capacity approaches a constant value when the antennas at the transmitter and receiver is sufficiently large.

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